KEY STAGE 2

In Years 3 and 4, children develop the basis of written methods by building their skills alongside a deep understanding of place value. They should use known addition/subtraction and multiplication/division facts to calculate efficiently and accurately, rather than relying on counting. Children use place value equipment to support their understanding, but not as a substitute for thinking.

Key language: partition, place value, tens, hundreds, thousands, column method, whole, part, equal groups, sharing, grouping, bar model

Addition and subtraction: In Year 3 especially, the column methods are built up gradually. Children will develop their understanding of how each stage of the calculation, including any exchanges, relates to place value. The example calculations chosen to introduce the stages of each method may often be more suited to a mental method. However, the examples and the progression of the steps have been chosen to help children develop their fluency in the process, alongside a deep understanding of the concepts and the numbers involved, so that they can apply these skills accurately and efficiently to later calculations. The class should be encouraged to compare mental and written methods for specific calculations, and children should be encouraged at every stage to make choices about which methods to apply.

In Year 4, the steps are shown without such fine detail, although children should continue to build their understanding with a secure basis in place value. In subtraction, children will need to develop their understanding of exchange as they may need to exchange across one or two columns. By the end of Year 4, children should have developed fluency in column methods alongside a deep understanding, which will allow them to progress confidently in upper Key Stage 2.

Multiplication and division: Children build a solid grounding in times-tables, understanding the multiplication and division facts in tandem. As such, they should be as confident knowing that 35 divided by 7 is 5 as knowing that 5 times 7 is 35. Children develop key skills to support multiplication methods: unitising, commutativity, and how to use partitioning effectively. Unitising allows children to use known facts to multiply and divide multiples of 10 and 100 efficiently. Commutativity gives children flexibility in applying known facts to calculations and problem solving. An understanding of partitioning allows children to extend their skills to multiplying and dividing 2- and 3-digit numbers by a single digit.

Children develop column methods to support multiplications in these cases.

For successful division, children will need to make choices about how to partition. For example, to divide 423 by 3, it is effective to partition 423 into 300, 120 and 3, as these can be divided by 3 using known facts.

Children will also need to understand the concept of remainder, in terms of a given calculation and in terms of the context of the problem. **Fractions:** Children develop the key concept of equivalent fractions, and link this with multiplying and dividing the numerators and denominators, as well as exploring the visual concept through fractions of shapes. Children learn how to find a fraction of an amount, and develop this with the aid of a bar model and other representations alongside.

in Year 3, children develop an understanding of how to add and subtract fractions with the same denominator and find complements to the whole. This is developed alongside an understanding of fractions as numbers, including fractions greater than 1. In Year 4, children begin to work with fractions greater than 1.

Decimals are introduced, as tenths in Year 3 and then as hundredths in Year 4. Children develop an understanding of decimals in terms of the relationship with fractions, with dividing by 10 and 100, and also with place value.

	Year 3		
	Concrete	Pictorial	Abstract
Year 3 Addition			
Understanding 100s	Understand the cardinality of 100, and the link with 10 tens. Use cubes to place into groups of 10 tens.	Unitise 100 and count in steps of 100.	Represent steps of 100 on a number line and a number track and count up to 1,000 and back to 0.
Understanding place value to 1,000	Unitise 100s, 10s and 1s to build 3-digit numbers.	Use equipment to represent numbers to 1,000. 200 240 241 Use a place value grid to support the structure of numbers to 1,000. Place value counters are used alongside other equipment. Children should understand how each counter represents a different unitised amount.	Represent the parts of numbers to 1,000 using a part-whole model. $215 = 200 + 10 + 5$ Recognise numbers to 1,000 represented on a number line, including those between intervals.
Adding 100s	Use known facts and unitising to add multiples of 100.	Use known facts and unitising to add multiples of 100.	Use known facts and unitising to add multiples of 100. Represent the addition on a number line.

	100 bricks bricks 100 bricks bricks 3 + 2 = 5 3 hundreds + 2 hundreds = 5 hundreds 300 + 200 = 500	3 + 4 = 7 3 hundreds + 4 hundreds = 7 hundreds 300 + 400 = 700	Use a part-whole model to support unitising. $3 + 2 = 5$ $300 + 200 = 500$
3-digit number + 1s, no exchange or bridging	Use number bonds to add the 1s. 214 + 4 = ? Now there are 4 + 4 ones in total. 4 + 4 = 8 214 + 4 = 218	Use number bonds to add the 1s. H	Understand the link with counting on. 245 + 4 245 246 247 248 249 250 Use number bonds to add the 1s and understand that this is more efficient and less prone to error. 245 + 4 = ? I will add the 1s. 5 + 4 = 9 So, 245 + 4 = 249
3-digit number + 1s with exchange	Understand that when the 1s sum to 10 or more, this requires an exchange of 10 ones for 1 ten. Children should explore this using unitised objects or physical apparatus.	Exchange 10 ones for 1 ten where needed. Use a place value grid to support the understanding.	Understand how to bridge by partitioning to the 1s to make the next 10.

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3-digit number + 10s, no exchange	Calculate mentally by forming the number bond for the 10s.	Calculate mentally by forming the number bond for the 10s. 351 + 30 = ?	Calculate mentally by forming the number bond for the 10s. $753 + 40$ I know that $5 + 4 = 9$ So, $50 + 40 = 90$

	234 + 50 There are 3 tens and 5 tens altogether. 3 + 5 = 8 In total there are 8 tens. 234 + 50 = 284	5 tens + 3 tens = 8 tens 351 + 30 = 381	753 + 40 = 793
3-digit number + 10s, with exchange	Understand the exchange of 10 tens for 1 hundred.	Add by exchanging 10 tens for 1 hundred. 184 + 20 = ? H T O SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS	Understand how the addition relates to counting on in 10s across 100. 184 + 20 = ? I can count in 10s 194 204 184 + 20 = 204 Use number bonds within 20 to support efficient mental calculations. 385 + 50 There are 8 tens and 5 tens. That is 13 tens. 385 + 50 = 300 + 130 + 5 385 + 50 = 435
3-digit number + 2-digit number	Use place value equipment to make and combine groups to model addition.	Use a place value grid to organise thinking and adding of 1s, then 10s.	Use the vertical column method to represent the addition. Children must understand how this relates to place value at each stage of the calculation.

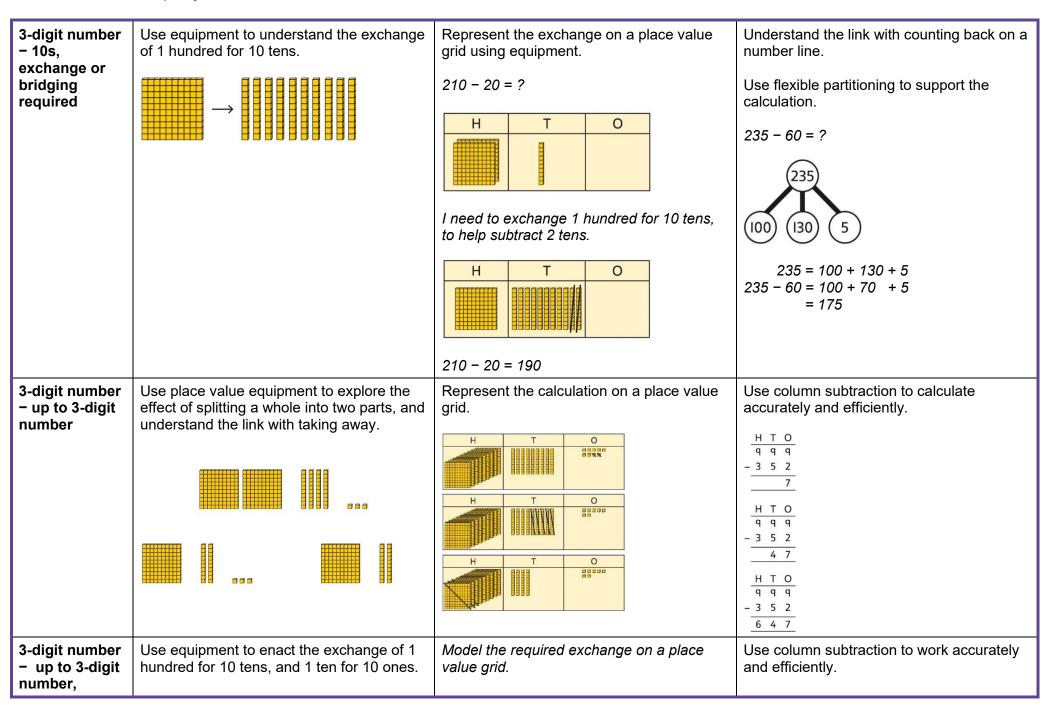
3-digit number + 2-digit number, exchange required	Use place value equipment to model addition and understand where exchange is required. Use place value counters to represent 154 + 72. Use this to decide if any exchange is required. There are 5 tens and 7 tens. That is 12 tens so I will exchange.	Represent the required exchange on a place value grid using equipment. 275 + 16 = ? H T O H T O 275 + 16 = 291 Note: In this example, a mental method may be more efficient. The numbers for the example calculation have been chosen to allow children to visualise the concept and see how the method relates to place value. Children should be encouraged at every stage to select methods that are accurate and efficient.	Use a column method with exchange. Children must understand how the method relates to place value at each stage of the calculation. H T O 2 7 5 + 1 6 - 1 6
3-digit number + 3-digit number, no exchange	Use place value equipment to make a representation of a calculation. This may or may not be structured in a place value grid. 326 + 541 is represented as:	Represent the place value grid with equipment to model the stages of column addition.	Use a column method to solve efficiently, using known bonds. Children must understand how this relates to place value at every stage of the calculation.

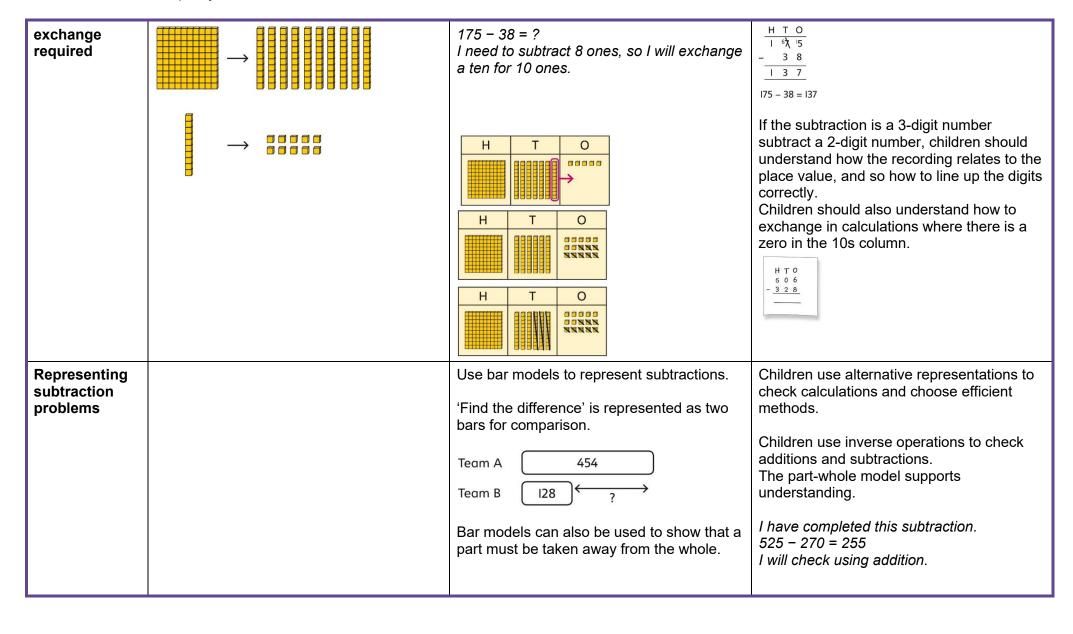
	3 2 6 5 4 I		
3-digit number + 3-digit number, exchange	Use place value equipment to enact the exchange required.	Model the stages of column addition using place value equipment on a place value grid.	Use column addition, ensuring understanding of place value at every stage of the calculation.
required	3000	H T O	H T O 2 6 + 2 7 3
	There are 13 ones. I will exchange 10 ones for 1 ten.	H T O	$ \frac{\begin{array}{c c} H & T & O \\ \hline I & 2 & 6 \\ + 2 & 1 & 7 \\ \hline \hline & 4 & 3 \\ \hline \end{array} $
		H T O	H T O 1 2 6 +2 1 7 3 4 3
			126 + 217 = 343 Note: Children should also study examples where exchange is required in more than one column, for example 185 + 318 = ?
Representing addition	Encourage children to use their own drawings and choices of place value	Children understand and create bar models to represent addition problems.	Use representations to support choices of appropriate methods.
problems, and selecting appropriate	equipment to represent problems with one or more steps.	275 + 99 = ?	? 275
methods	These representations will help them to select appropriate methods.	275	I will add 100, then subtract 1 to find the solution.
		275 + 99 = 374	

	I		
			128 + 105 + 83 = ? I need to add three numbers.
			128 + 105 = 233
			128 105 83
			233 83
Year 3 Subtraction			
Subtracting 100s	Use known facts and unitising to subtract multiples of 100.	Use known facts and unitising to subtract multiples of 100.	Understand the link with counting back in 100s.
	100 bricks 100 bricks 100 bricks 5 - 2 = 3 500 - 200 = 300	4 - 2 = 2 400 - 200 = 200	Use known facts and unitising as efficient and accurate methods. I know that $7 - 4 = 3$. Therefore, I know that $700 - 400 = 300$.
3-digit number - 1s, no exchange	Use number bonds to subtract the 1s. 214 - 3 = ?	Use number bonds to subtract the 1s.	Understand the link with counting back using a number line. Use known number bonds to calculate mentally. 476 - 4 = ?
		319 - 4 = ?	(476) (400) (70) (6)

381 - 10 = 371

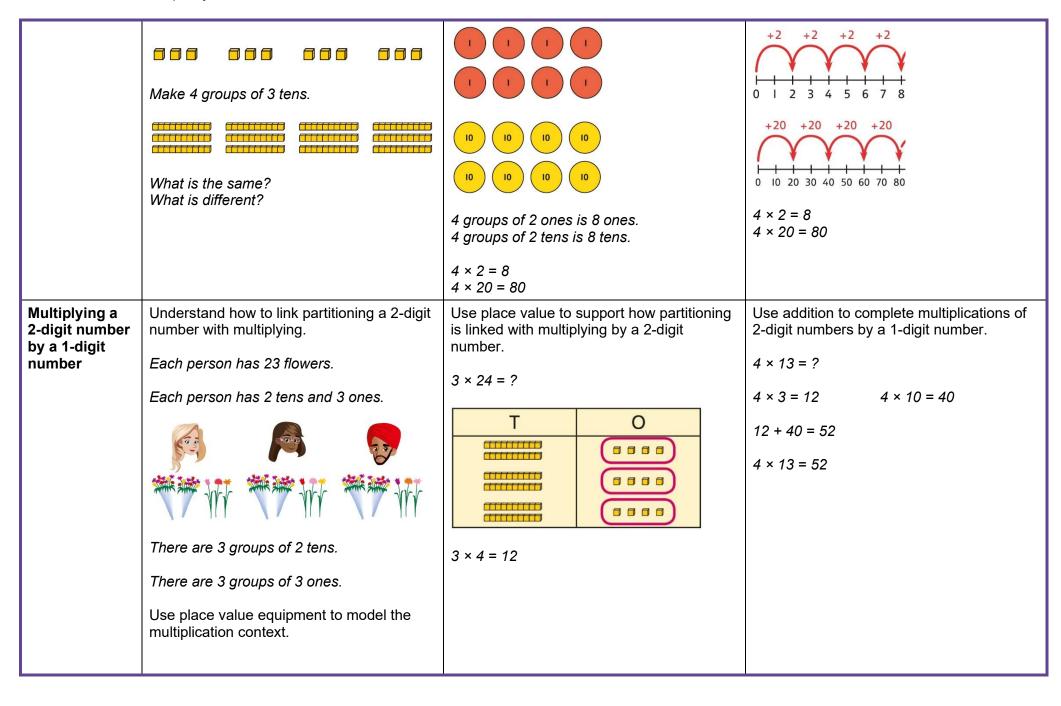
	,		
	100 tollies 10 LOLLIES 4 - 3 = 1 214 - 3 = 211	9 - 4 = 5 319 - 4 = 315	6 - 4 = 2 476 - 4 = 472
3-digit number – 1s, exchange or bridging required	Understand why an exchange is necessary by exploring why 1 ten must be exchanged. Use place value equipment.	Represent the required exchange on a place value grid. 151 - 6 = ? H T O H T O	Calculate mentally by using known bonds. 151 - 6 = ? 151 - 1 - 5 = 145
3-digit number − 10s, no exchange	Subtract the 10s using known bonds. 381 - 10 = ? 8 tens with 1 removed is 7 tens.	Subtract the 10s using known bonds. H T O 8 tens - 1 ten = 7 tens 381 - 10 = 371	Use known bonds to subtract the 10s mentally. 372 - 50 = ? 70 - 50 = 20 So, 372 - 50 = 322

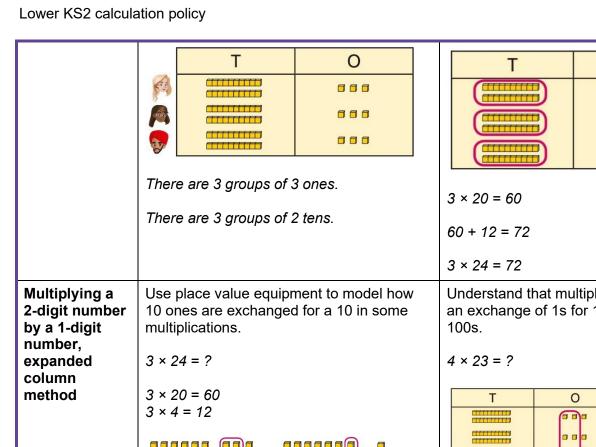




Year 3 Multiplication			(525) (270) (255) H T O 2 7 0 + 2 5 5 5 2 5
Understanding equal grouping and repeated addition	Children continue to build understanding of equal groups and the relationship with repeated addition. They recognise both examples and nonexamples using objects. Children recognise that arrays can be used to model commutative multiplications. I can see 3 groups of 8. I can see 8 groups of 3.	Children recognise that arrays demonstrate commutativity. This is 3 groups of 4. This is 4 groups of 3.	Children understand the link between repeated addition and multiplication. $ \begin{array}{cccccccccccccccccccccccccccccccccc$
Using commutativity to support	Understand how to use times-tables facts flexibly.	Understand how times-table facts relate to commutativity.	Understand how times-table facts relate to commutativity.

understanding of the times- tables	There are 6 groups of 4 pens. There are 4 groups of 6 bread rolls. I can use 6 × 4 = 24 to work out both totals.	6 × 4 = 24 4 × 6 = 24	I need to work out 4 groups of 7. I know that 7 × 4 = 28 so, I know that 4 groups of 7 = 28 and 7 groups of 4 = 28.
Understanding and using ×3, ×2, ×4 and ×8 tables.	Children learn the times-tables as 'groups of', but apply their knowledge of commutativity. I can use the ×3 table to work out how many keys. I can also use the ×3 table to work out how many batteries.	Children understand how the ×2, ×4 and ×8 tables are related through repeated doubling.	Children understand the relationship between related multiplication and division facts in known times-tables. $2 \times 5 = 10$ $5 \times 2 = 10$ $10 \div 5 = 2$ $10 \div 2 = 5$
Using known facts to multiply 10s, for example 3 × 40	Explore the relationship between known times-tables and multiples of 10 using place value equipment. Make 4 groups of 3 ones.	Understand how unitising 10s supports multiplying by multiples of 10.	Understand how to use known times-tables to multiply multiples of 10.





+ 12

 $3 \times 24 = 60 + 12$

 $3 \times 24 = 70 + 2$ $3 \times 24 = 72$

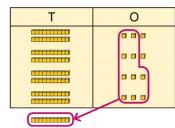
Understand that multiplications may require an exchange of 1s for 10s, and also 10s for

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	0 0
$4 \times 23 = 92$	

Children may write calculations in expanded column form, but must understand the link with place value and exchange.

Children are encouraged to write the expanded parts of the calculation separately.

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 6×5 6×10

$$5 \times 28 = ?$$

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Year 3 Division			
Using times- tables knowledge to divide	Use knowledge of known times-tables to calculate divisions. 24 divided into groups of 8. There are 3 groups of 8.	Use knowledge of known times-tables to calculate divisions.	Use knowledge of known times-tables to calculate divisions. I need to work out 30 shared between 5. I know that $6 \times 5 = 30$ so I know that $30 \div 5 = 6$. A bar model may represent the relationship between sharing and grouping. $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$

Understanding remainders Using known facts to divide multiples of 10	Use equipment to understand that a remainder occurs when a set of objects cannot be divided equally any further. There are 13 sticks in total. There are 3 groups of 4, with 1 remainder. Use place value equipment to understand how to divide by unitising. Make 6 ones divided by 3. Now make 6 tens divided by 3.	Use images to explain remainders. 22 ÷ 5 = 4 remainder 2 Divide multiples of 10 by unitising. 12 tens shared into 3 equal groups. 4 tens in each group.	24 ÷ 8 = 3 H8 H8 H8 H8 H8 H8 H8 H8 H8
2-digit number divided by 1-digit number, no remainders	What is the same? What is different? Children explore dividing 2-digit numbers by using place value equipment.	Children explore which partitions support particular divisions.	Children partition a number into 10s and 1s to divide where appropriate.

	48 ÷ 2 = ? First divide the 10s. Then divide the 1s.	I need to partition 42 differently to divide by 3. Integration 42 differently to divide by 3. $42 = 30 + 12$ $42 \div 3 = 14$	$60 \div 2 = 30$ $8 \div 2 = 4$ $30 + 4 = 34$ $68 \div 2 = 34$ Children partition flexibly to divide where appropriate. $42 \div 3 = ?$ $42 = 40 + 2$ I need to partition 42 differently to divide by 3. $42 = 30 + 12$ $30 \div 3 = 10$ $12 \div 3 = 4$ $10 + 4 = 14$ $42 \div 3 = 14$
2-digit number divided by 1-digit number, with remainders	Use place value equipment to understand the concept of remainder. Make 29 from place value equipment. Share it into 2 equal groups. There are two groups of 14 and 1 remainder.	Use place value equipment to understand the concept of remainder in division. 29 ÷ 2 = ? 29 ÷ 2 = 14 remainder 1	Partition to divide, understanding the remainder in context. 67 children try to make 5 equal lines. 67 = 50 + 17 50 ÷ 5 = 10 17 ÷ 5 = 3 remainder 2 67 ÷ 5 = 13 remainder 2 There are 13 children in each line and 2 children left out.

Year 4					
	Concrete	Pictorial	Abstract		
Year 4 Addition					
Understanding numbers to 10,000	Use place value equipment to understand the place value of 4-digit numbers. 4 thousands equal 4,000. 1 thousand is 10 hundreds.	Represent numbers using place value counters once children understand the relationship between 1,000s and 100s. 2,000 + 500 + 40 + 2 = 2,542	Understand partitioning of 4-digit numbers, including numbers with digits of 0. $5,000 + 60 + 8 = 5,068$ Understand and read 4-digit numbers on a number line.		
Choosing mental methods where appropriate	Use unitising and known facts to support mental calculations. Make 1,405 from place value equipment. Add 2,000. Now add the 1,000s. 1 thousand + 2 thousands = 3 thousands 1,405 + 2,000 = 3,405	Use unitising and known facts to support mental calculations. Th H T O Th H T O Th A T O Th	Use unitising and known facts to support mental calculations. 4,256 + 300 = ? 2 + 3 = 5		

Column addition with exchange

Use place value equipment on a place value grid to organise thinking.

Ensure that children understand how the columns relate to place value and what to do if the numbers are not all 4-digit numbers.

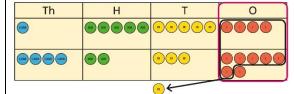
Use equipment.to show 1,905 + 775.

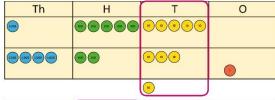
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Why have only three columns been used for the second row? Why is the Thousands box empty?

Which columns will total 10 or more?

Use place value equipment to model required exchanges.



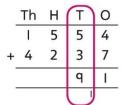


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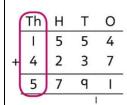
Th	Н	Т	0
1,000	100 100 100 000	0 0 0 0 0	
1,500 1,500 1,000 1,000	110 110	9 0 0	0

Include examples that exchange in more than one column.

Use a column method to add, including exchanges.



112	Th	H	Т	0
	1	5	5	4
+	4	2	3	7
		7	9	1

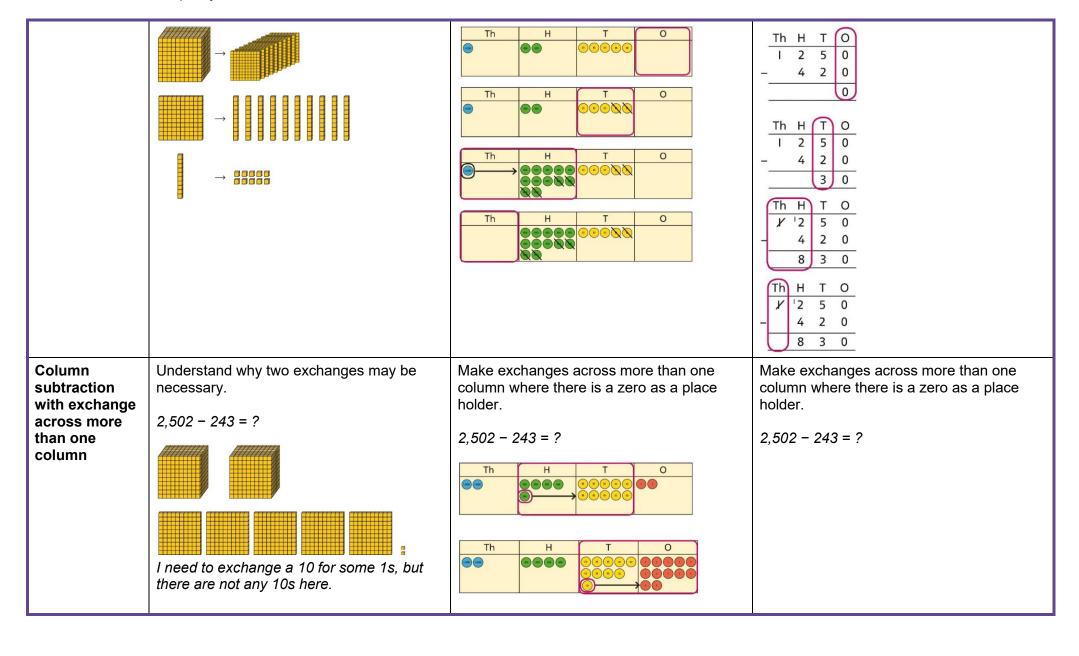


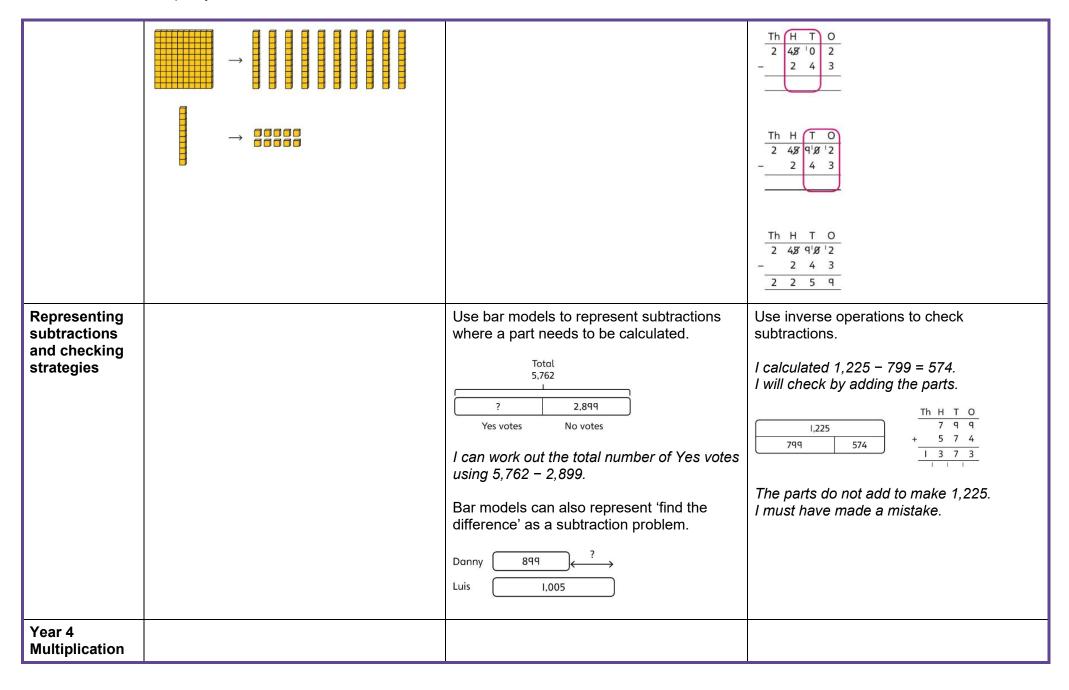
Include examples that exchange in more than one column.

Representing additions and checking strategies

Bar models may be used to represent additions in problem contexts, and to justify mental methods where appropriate. Use rounding and estimating on a number line to check the reasonableness of an addition.

Year 4		The Heat To 7 9 9 1,373	912 + 6,149 = ? I used rounding to work out that the answer should be approximately 1,000 + 6,000 = 7,000.
Choosing mental methods where appropriate	Use place value equipment to justify mental methods. What number will be left if we take away 300?	Use place value grids to support mental methods where appropriate. Th	Use knowledge of place value and unitising to subtract mentally where appropriate. 3,501 - 2,000 3 thousands - 2 thousands = 1 thousand 3,501 - 2,000 = 1,501
Column subtraction with exchange	Understand why exchange of a 1,000 for 100s, a 100 for 10s, or a 10 for 1s may be necessary.	Represent place value equipment on a place value grid to subtract, including exchanges where needed.	Use column subtraction, with understanding of the place value of any exchange required.





Multiplying by multiples of 10 and 100	Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.	Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.	Use known facts and understanding of place value and commutativity to multiply mentally.
	3 groups of 4 ones is 12 ones. 3 groups of 4 tens is 12 tens. 3 groups of 4 hundreds is 12 hundreds.	3 × 4 = 12 3 × 40 = 120 3 × 400 = 1,200	$4 \times 7 = 28$ $4 \times 70 = 280$ $40 \times 7 = 280$ $4 \times 700 = 2,800$ $400 \times 7 = 2,800$
Understanding times-tables up to 12 × 12	Understand the special cases of multiplying by 1 and 0.	Represent the relationship between the ×9 table and the ×10 table.	Understand how times-tables relate to counting patterns.
up το 12 * 12	$5 \times 1 = 5$ $5 \times 0 = 0$	Represent the ×11 table and ×12 tables in relation to the ×10 table. $2 \times 11 = 20 + 2$ $3 \times 11 = 30 + 3$ $4 \times 11 = 40 + 4$	Understand links between the $\times 3$ table, $\times 6$ table and $\times 9$ table 5×6 is double 5×3 $\times 5$ table and $\times 6$ table I know that $7 \times 5 = 35$ so I know that $7 \times 6 = 35 + 7$. $\times 5$ table and $\times 7$ table $3 \times 7 = 3 \times 5 + 3 \times 2$ $3 \times 5 \times $
Understanding and using partitioning in multiplication	Make multiplications by partitioning. 4 × 12 is 4 groups of 10 and 4 groups of 2.	Understand how multiplication and partitioning are related through addition.	Use partitioning to multiply 2-digit numbers by a single digit. 18 × 6 = ?

	4 × 12 = 40 + 8	4 × 3 = 12 4 × 3 = 12 4 × 5 = 20 12 + 20 = 32 4 × 8 = 32	$ \begin{array}{c} $
Column multiplication for 2- and 3-digit numbers multiplied by a single digit	Use place value equipment to make multiplications. Make 4 × 136 using equipment. I can work out how many 1s, 10s and 100s. There are 4 × 6 ones 24 ones There are 4 × 3 tens 12 tens There are 4 × 1 hundreds 4 hundreds 24 + 120 + 400 = 544	Use place value equipment alongside a column method for multiplication of up to 3-digit numbers by a single digit. 3	Use the formal column method for up to 3-digit numbers multiplied by a single digit. 3
Multiplying more than two numbers	Represent situations by multiplying three numbers together.	Understand that commutativity can be used to multiply in different orders. 2 × 6 × 10 = 120	Use knowledge of factors to simplify some multiplications. 24 × 5 = 12 × 2 × 5

Year 4	Each sheet has 2×5 stickers. There are 3 sheets. There are $5 \times 2 \times 3$ stickers in total. $5 \times 2 \times 3 = 30$ $10 \times 3 = 30$	12 × 10 = 120 10 × 6 × 2 = 120 60 × 2 = 120	12 × 2 × 5 =
Division Understanding the relationship between multiplication and division, including times-tables	Use objects to explore families of multiplication and division facts.	Represent divisions using an array.	Understand families of related multiplication and division facts. I know that $5 \times 7 = 35$ so I know all these facts: $5 \times 7 = 35$ $7 \times 5 = 35$ $35 = 5 \times 7$ $35 = 7 \times 5$ $35 \div 5 = 7$ $35 \div 7 = 5$ $7 = 35 \div 5$ $5 = 35 \div 7$
Dividing multiples of 10 and 100 by a single digit	Use place value equipment to understand how to use unitising to divide.	Represent divisions using place value equipment.	Use known facts to divide 10s and 100s by a single digit. $15 \div 3 = 5$ $150 \div 3 = 50$

	8 ones divided into 2 equal groups 4 ones in each group 8 tens divided into 2 equal groups	$9 \div 3 =$ 1	1500 ÷ 3 = 500
	4 tens in each group 8 hundreds divided into 2 equal groups 4 hundreds in each group	9 tens divided by 3 is 3 tens. 9 hundreds divided by 3 is 3 hundreds.	
Dividing 2-digit and 3-digit	Partition into 10s and 1s to divide where appropriate.	Partition into 100s, 10s and 1s using Base 10 equipment to divide where appropriate.	Partition into 100s, 10s and 1s using a part- whole model to divide where appropriate.
numbers by a single digit by partitioning into 100s, 10s and 1s	39 ÷ 3 = ?	39 ÷ 3 = ?	142 ÷ 2 = ?
	3 × 10 = 30 3 × 3 = 9	3 groups of I ten 3 groups of 3 ones	100 40 6 100 ÷ 2 = 6 ÷ 2 =
	39 = 30 + 9	39 = 30 + 9	100 ÷ 2 = 50
	30 ÷ 3 = 10	30 ÷ 3 = 10	40 ÷ 2 = 20 6 ÷ 2 = 3
	9 ÷ 3 = 3 39 ÷ 3 = 13	9 ÷ 3 = 3 39 ÷ 3 = 13	50 + 20 + 3 = 73 142 ÷ 2 = 73
Dividing 2-digit and 3-digit numbers by a	Use place value equipment to explore why different partitions are needed.	Represent how to partition flexibly where needed.	Make decisions about appropriate partitioning based on the division required.
single digit,	42 ÷ 3 = ?	84 ÷ 7 = ?	72 72 72
using flexible partitioning	I will split it into 30 and 12, so that I can divide by 3 more easily.	I will partition into 70 and 14 because I am dividing by 7.	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

